Assessment of Mathematical Pedagogical Content Knowledge of Early Childhood Education Pre-Service Teachers in Southwestern Nigeria

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Abstract
This study examined early childhood education pre-service teachers' mathematics pedagogical content knowledge (MPCK) in South-western, Nigeria. Mixed method research approach was used in the study. Final year pre-service teachers (n = 120) of Early Childhood Education in College of Education who were on teaching practice programme were purposively selected for the study. Direct observations of the pre-service teachers (PSTs) during engagement with children as well as interview were used to generate data for the study. Means, percentages and standard deviation as well as thematic analysis were used to analyse the data. The study revealed that pre-service teachers performed below expectation *on the major five dimensions of assessment (i.e. Content knowledge, instructional practices, classroom environment and management, interaction and communication, and assessment practices). Insufficient courses in early mathematics, rare practical opportunities and mentoring were the major challenges confronting the PSTs. It was recommended that more attention is given to content knowledge of mathematics pedagogy in the curriculum of pre-service teachers to enhance comprehension of early mathematics for improved performance. Additionally, quality mentoring opportunities should be provided to PSTs before getting into the teaching field.*

Keywords: Early childhood pre-service teachers, Early mathematics education, Pedagogical content knowledge

Introduction

Mathematics play a critical role in early childhood education, offering young learners the skills they need to navigate and understand their surroundings. The early education is essential for providing young children with a strong foundation in mathematics and other core skills. Through mathematics, children can develop important problem-solving, reasoning and analytical skills that are applicable across various areas of life. Early mathematics in early childhood education has been found to support cognitive development, foster critical thinking and promote a love for

learning (Dearing *et al.,* 2009). Quality early mathematics education positively affects children's love for mathematics.and achievement (Zhang *et al.,* 2020). Moreover, early mathematics education foster a positive attitude toward learning and sets the stage for future success in mathematics as well as other subjects (Aunio and Niemivirta, 2010; Duncan *et al.,*. 2007; Krajewski and Schneider, 2009). Children who develop confidence in their mathematics abilities early are more likely to approach challenges with enthusiasm and resilience. This confidence can have lasting effects on their academic trajectories and career prospects On the other hand, weak mathematical skills in early childhood years are likely to predict weak mathematics performance in later school years (Reikerås & Salomonsen, 2019).

Teachers with adequate knowledge and skills in early mathematics teaching can contribute meaningfully to improving children's mathematics skills (Keuch & Brandt, 2020; Bruns *et al.,* 2020). When children receive high quality mathematics instruction from an early age, they have a better chance of succeeding academically and reaching their full potentials in life. Central to providing high quality mathematics instruction in early childhood education is the Mathematical Pedagogical Content Knowledge (MPCK) of the teachers, which is regarded as the ability of the teacher to seamlessly integrate mathematics content knowledge with pedagogical strategies (McCray, 2008; Zhang, 2015) Mathematical pedagogical content knowledge for pre-school teachers is not limited to knowing mathematical concepts but, involves deep understanding of how to effectively teach mathematical concepts using effective teaching strategies, appropriate assessment practices as well as differentiation of learning to accommodate for diverse learners.

Studies have highlighted the significance of MPCK in shaping teachers' confidence and competence in delivering high quality mathematics instruction (Chen, 2012; McCray, 2008). Literature has revealed that teachers with strong MPCK are well equipped to design effective lessons that are developmentally appropriate, engaging and aligned with curricular standards (Cerezci, 2020). They can employ a variety of instructional strategies to make mathematical abstract concepts more concrete, understandable and enjoyable for the children. In addition, teachers with high MPCK are more skilled at identifying and addressing students' misconceptions about mathematical concepts. Early intervention to correct such misconceptions can prevent students from developing persistent learning gaps and set them on the path to a solid mathematical foundation. Moreover, research emphasizes that MPCK contributes to teachers' ability to create

positive classroom environment, where children can feel comfortable to explore mathematical concepts. Such atmospheres enhance children's confidence and motivation to engage with mathematics. Apart from this, differentiation is another key aspect of high quality Mathematics instruction that relies on teachers' MPCK. By adapting instruction to meet the diverse needs of students, teachers can provide individualized support and challenges, helping all children succeed, regardless of their starting points.

Regrettably, many studies have observed that most pre-school teachers possess low level of mathematical pedagogical content knowledge (Cerezci, 2019; Korkmaz & Şahin, 2019; Li, 2021; Torbeyns *et al.*, 2020). Additionally, studies have equally reported a feeling of teachers' inadequacy in teaching mathematics (Lee, 2010; Sheridan *et al.,* 2011). While substantial research has explored MPCK in different context (Lee, 2017; McCray & Chen, 2012 Figueiredo*et al., 2018*), there is a noticeable gap in understanding how pre-service teachers (PSTs) in early childhood education in Nigeria apply this specialized knowledge. This study addressed this gap.

Understanding the level of MPCK early childhood pre-service teachers is essential for targeted support and professional development. This identified gap in mathematics pedagogical content knowledge can pose a substantive impediment to the comprehensive development of children's mathematical cognition and achievement in Nigeria, if not given deserved attention. Evidently, the expectation that teachers will transmit knowledge, skills, and attitudes to students in the absence of their own proficiency therein is untenable, as it disregards the critical role of teachers' competence in facilitating effective learning outcomes. Focusing on determining the level of MPCK of PSTs is therefore imperative to understand the type of support that will be most required.

There is no doubt that teacher preparation programme can significantly enhance the development of MPCK of pre-service teachers. Effective teacher preparation programmes provide opportunities for pre-service teachers to be deeply engaged with mathematical contents and instructional strategies fostering a solid understanding of both. They also provide practical experience opportunities such as classroom observations, practicum placement and student teaching practice which allow the PSTs to gather real world experience to enhance their MPCK. Effective teacher preparation can therefore pave way for quality early mathematics teaching that sets children on the path to mathematical success. Pre-service teachers with strong MPCK are likely to feel more confident and competent in their ability to teach mathematics (Fuentes-Abeledo *et al.,*2020).This

confidence can translate into more positive interactions with the children and a greater willingness to try innovative teaching methods.

While many studies highlight the significance of mathematical pedagogical content knowledge (MPCK) in shaping pre-service teachers' confidence and competence in delivering high quality mathematics instruction. . There is limited research on the extent and effectiveness of pre-service teachers' MPCK in the Nigerian context. In particular, there is a need to understand how well-prepared pre-service teachers are in teaching mathematics to young learners, what challenges they face and, how these factors impact their ability to deliver high-quality, engaging mathematics instruction. This study sought to investigate the current state of early childhood pre-service teachers' MPCK and identify potential areas for improvement to enhance mathematics education for young children in Nigeria. This research also provides valuable insights into the effectiveness of teacher preparation programs and informs on future improvements.

In early childhood mathematics education, several existing theories and frameworks provide valuable insights into effective teaching practices and the development of pedagogical content knowledge (PCK). One key theory is Constructivism, which posits that children construct their understanding of mathematics through active engagement with concepts. This approach encourages teachers to create learning environments where students can explore, question and apply mathematical ideas, fostering deeper understanding and retention. Additionally, Lev Vygotsky's concept of the Zone of Proximal Development highlighted the importance of scaffolding in teaching, suggesting that teachers guide students through challenging tasks while gradually allowing them more autonomy as they become more proficient in mathematical concepts.

Another important framework is Cognitively Guided Instruction (CGI), which focused on understanding children's intuitive mathematical thinking and building instruction around their existing knowledge. This approach promotes deeper conceptual understanding by acknowledging and leveraging students' natural abilities and ways of thinking. Similarly, the Teaching for Understanding framework, developed by the Harvard Graduate School of Education, advocated for teaching that goes beyond rote memorization and emphasizes the application of mathematical concepts in real life contexts. These theories and frameworks guided the scope of MPCK as examined in this study. These theories are essential for Early childhood teachers in developing

effective and designing instructions that will foster a strong foundation in mathematics for young learners. Incorporating these approaches, teachers can better support students' mathematical development and cultivate a lifelong interest in, and appreciation for mathematics.

Research Questions

1. What is the current level of mathematical pedagogical content knowledge (MPCK) of early childhood pre-service teachers?

2. What challenges do early childhood pre-service teachers face in adapting MPKC during their teaching practices?

Methodology

This study adopted mixed method research design to explore early childhood education pre-service teachers' mathematical pedagogical content knowledge during their final compulsory teaching practices in the South West Nigeria. A sequential explanatory design of mixed research design was utilized in the study, where quantitative and qualitative data were collected sequentially. Quantitative data was collected to determine the extent to which the PSTs adapted MPCK during their teaching practices, while qualitative data were collected through interviews of the PSTs to get insight into the challenges that PSTs face in developing and applying MPCK in early childhood education. The qualitative data presents in-depth understanding of quantitative findings. The design helped to generate relevant data to provide answers to the research questions.

Early Childhood and Care Education (ECCE) Students of Colleges of Education undergo a compulsory professional teaching practice for a whole first semester in their third year, during which they are posted to various early childhood schools within the States in which their Colleges are located. The PSTs are required to plan and implement lessons from different aspects of the National Curriculum in Early Childhood Education in Nigeria during the period. The PSTs are required to provide full-time dedication to this process. This study included 120 (14 males and 106 females) Early Childhood Education PSTs who were purposively selected across 12 public Colleges of Education in South West Nigeria. The PSTs must be willing to participate in the study and must be undergoing their final professional teaching practice in early childhood schools before they were included in the study. Their ages varied slightly between 18 and 23 years. Informed

consent was obtained from the participants and their teaching practice schools before the commencement of the study.

The data in this study were collected by the researcher and 10 research assistants who had been trained with the instrument used for the study. Classroom observations and interviews were used to collect data for this study.

Classroom Observation

This study adopted direct observation of the activities and interactions of the pre-service teachers with the children during mathematics instruction to assess how effectively they applied MPCK in real classroom settings. This approach has been found to provide great insight into a teacher's ability to perform pedagogical content knowledge (Shanahan & Tochelli, 2014). Physical observation of mathematics classroom activities was carried out for 30 minutes in a day for a period of 21 days on each participating pre-service teacher, to capture representative sample of their MPCK practices in their respective teaching practice schools. Video coverage of the mathematics lessons was also taken and later coded to re-validate the documentation. The observers were trained on goals and objectives of the observation, specific behaviours related to each dimension of MPCK and how to accurately assess each of the dimensions of MPCK for one week and were certified effective before they were involved in the study.

The instrument titled: Early Mathematical Pedagogical Content Knowledge Assessment Scale for Pre-service Teachers (EMPCKASPT) was designed to assess various key aspects of MPCK in early childhood education classroom to provide valuable insights into the strengths and areas for growth in the pedagogical content knowledge and teaching practices of PST's. The instrument is composed of 50 items consisting of five major dimensions of MPCK which are Mathematical Content Knowledge (10 items), Instructional Practices (15 items); Interactions and Communication (10 items), Classroom Environment and Management (7 items), Assessment and Feedback (8 items).Within each category, specific criteria and behaviors were outlined to guide observers in evaluating teachers' abilities to effectively integrate subject matter knowledge with pedagogical strategies to support children's mathematics learning. The instrument was measured on 4 point rating scale, ranging from Exceptional (4); Proficient (3); Basic (2); Unsatisfactory (1). The instrument was designed by the researcher in line with literature on effective components of

MPCK in early childhood education. Two experts in early childhood education reviewed and certified the instrument for face, content and construct validity. The instrument was tested for reliability using inter-rater method and a value of 0.85 was obtained. This value showed that the instrument was reliable in the context of the research.

Table 1: Mean Intervals and Description of Level of Early Mathematical Pedagogical Content Knowledge (EMPCK)

Rating	Mean Range	Performance	Level of Competence
4	3.5-4.0	High	High
3	3.4-2,8	Average	Average
2	2.9- 2,.0	Low	Low
1	1.9 - 1.0	Very low	*Very low*

Interview

Semi-structured interviews were conducted with 20 PSTs who were randomly selected from 120 participants involved in the study to explore their understanding of MPCK as well as to identify challenges that confront them in adapting MPCK into their teaching practices. The interviews were conducted at the last day of classroom observations and lasted for average of 35 minutes for each of the participants. The participants responded to five open-ended questions that encouraged detailed responses of the PSTs on their MPCK.

The interviews were audio recorded. Pseudonyms were used in order to protect participants' privacy. The recorded responses of the PSTs were transcribed verbatim to capture all the data in written form. Thematic analysis was used to analyse the interview responses. Patterns or trends across participants' responses, were identified in relation to the research questions. The data were coded through data reduction and summarization where similar responses were put together to generate themes and sub-themes.

Results

The results are presented in Tables 2 and 3 with the related analysis and interpretation in line with the research questions posed in the study. Table 2 shows the levels to which PSTs adapted MPCK in their teaching practices while Table 3 shows challenges confronting PSTs in adapting MPCK.

Table 2: Summary of the Extent to which Pre-service Early Childhood Teachers Adapt Mathematical Pedagogical Content Knowledge in their Teaching Practices

Category and Dimensions	Mean	SD	Description
Mathematical Content Knowledge	2.1	0.62	Low
Instructional Practices	1.5	0.70	Poor
Classroom Environment and Management	1.3	0.72	Poor
Interaction and Communication	1.2	0.65	Poor
Assessment Strategies	1.1	0.78	Poor
Weighted Average =1.44 (36.00%)			

Overall, results of the observations indicate that the level of MPCK of PSTs is poor with weighted average of 1.44 (36.00%). The PSTs performed low in all the dimensions of MPCK examined in the study. As shown in Table 2, for the mathematical content knowledge dimension, (M = 2.1, SD = 0.62), the result shows that the PSTs were just at the basic level. Although, it appears as the only dimension in which the PSTs did best, notwithstanding, the performance is just barely at the basic level while the PSTs performed the least in the dimension of Assessment Strategies (M = 1.1; SD = 0.78),

Table 3: Major Themes and Sub-Themes that emerged from Interviews on the Challenges confronting Early Childhood Pre-Service Teachers in Adapting Mathematical Pedagogical Content Knowledge

Themes	Sub-themes
Limited Exposure to MPCK	• Limited practical experiences • Poor mentoring opportunities • Few courses on early years mathematics • Lack of confidence
Insufficient Resources	• Limited learning materials • Rigid time table and time constraint

The analysis of interview responses of the PSTs showed two main themes which were: Limited exposure to MPCK and Insufficient resources, with the associated sub-themes that are presented in Table 3.

Limited Exposure to MPCK

As shown earlier in Table 2, the responses also show a clear demonstration of poor understanding of domains of MPCK that are required for effective mathematics education in early childhood education. The PSTs attributed this challenge to the limited experience with courses relating to MPCK in their training programme. For example, they reported that they had just one course on mathematics in early years. The PSTs also reiterated that limited time was allocated to practical teaching experiences which could have provided ample opportunities for hands-on experiences, guided practice and reflective activities that could assist them to apply what they have learnt in school in their teaching practices.

Some of the PSTs made the following statements in this regard:

> *"I have problems practicing some of the strategies of teaching methods that we were taught in school with the children as I do not really understand how to implement it'. PSTs1*

"Some of the courses we have taken in school included some component of MPCK, but I do not have the confidence to implement them in my teaching because, the courses were taught in more theoretical ways, so I find it difficult to implement them in real classroom settings" PSTs3

Furthermore, they submitted that they do not sufficiently enjoy mentoring opportunities, they stressed that they lacked opportunity to work closely with mentors who have track records of effective practice whom they can observe in action, learn from their knowledge and wealth of experience in early mathematics teaching. Some of PSTs provided the following statements:

"We have little time to undergo practical in my school as we most of the time concentrate on how to pass our examinations than how to effectively engage children in real classroom setting" PSTs 12

"I wish to have more of guided practice in the real early childhood classroom, which I believe can help me refine my skills as well as boost my confidence in implementing effective mathematics experience with the children" PSTs 8

"I have had little opportunity to be mentored by experts who can provide me with necessary support and experience to implement effective mathematics lessons with the children in the real world teaching environment" PSTs 14

Insufficient Resources

The PSTs acknowledged during interview that lack of learning resources was one of the major hindrances to adapting MPCK in their teaching practices. The PSTs reported that most of the schools they are posted to carry out their teaching practices lacked essential learning facilities, required to implement innovative teaching methods in early childhood classroom setting. They believed the deficit in learning resources have limited opportunities for the provision of hands-on experiences in the schools. For example, the PSTs reported that there are few learning materials

such as blocks, counters and number charts, visual aids and games, in most of the schools they are currently interning.

> *"In this school, we lack essential resources that can be used to plan developmentally appropriate mathematics lesson, this is one of the challenges that we encounter and is affecting practice. PSTs 9*

> *"I teach using the formal methods that the main teachers' uses' PSTs 13*

Discussion of Findings

Level of MPCK of PSTs

The goal of this study was to the determine the level of MPCK of early childhood PSTs during their teaching practices as well as challenges confronting them in adapting the knowledge to teaching mathematics to children in early childhood settings. As shown in Table 2, it can be seen that the overall weighted mean for MPCK of pre-service early childhood education teachers was poor. This performance could have emanated from limitation in the initial teacher preparation and personal experience which may have had a negative effect on their performance. The findings from the interviews conducted with the pre-service teachers clearly affirm that they possess a limited understanding of MPCK, as many of the responses of the PSTs indicated a narrow understanding of what constitutes MPCK. The findings of poor MPCK in this study is consistent with the observations of Markovits & Patkin, (2020) and Korkmaz & Şahin, 2019, who found that a great number of pre-teachers lacked mathematical content knowledge. Many studies conducted on preschool teachers arrived at similar conclusions (Chen *et al*., 2014; Li, 2021; Torbeyns *et al*., 2020).

This lack of understanding of mathematics pedagogical content knowledge of PSTs is of great concern to providing quality mathematics for children in the early years, as it can hinder their ability to effectively teach mathematical concepts to young children, leading to gaps in their understanding and skill development. Furthermore, poor level of MPCK can limit teachers' ability to identify and address individual learning needs, potentially leaving some children

behind in their mathematical understanding. This view is in line with reports of Keuch and Brandt, and Bruns *et al*., (2020) that PSTs knowledge of mathematics in early childhood education affected their readiness to utilize the content in ways that facilitate children's Mathematical learning.

The Challenges Confronting Early Childhood Pre-Service Teachers in Adapting MPCK

As shown in Table 3, the findings from the interview revealed that one of the major challenges of PSTs in adapting MPCK to their teaching practices is limited exposure to various dimensions of providing effective mathematical experiences for the children. One significant aspect of this is the limited number of courses addressing early mathematics in Colleges of Education programs in Nigeria. Studies have reported similar observations that most of the early childhood teacher preparation programs do not offer courses in early mathematics teaching and learning which are capable of equipping pre-service teachers with necessary content and pedagogical knowledge in mathematics required to provide quality mathematics education (Cerezci, 2019 ; Ginsburg, *et al*. 2008; National Research Council, 2009). The participants reported that only one course is dedicated to early years' mathematics in their programs. This observation implies that their MPCK was not sufficiently addressed in their course work. This may largely be responsible for the poor performance of PSTs on all the dimensions of MPCK that were examined in this study.

Moreover, the PSTs believed that the manner in which the course was administered was poor, as they reported that little attention was paid to its practicality in the real classroom setting. The PSTs admitted that there was an unspoken priority on passing test and examination over practical activities as they claimed that more time were allocated to theory than practical. Cerezci (2019) made similar observation in a study where it was reported that PSTs were exposed to insufficient practical hands-on training. Providing comprehensive courses and training is therefore essential to promote rich, quality and relevant mathematical experiences for children.

Conclusion

This study found that pre-service early childhood teachers performed very low on all the five major dimension of mathematical pedagogical knowledge examined, including content knowledge, instructional practices, classroom management and environment, interaction and communication

and assessment practices. Additionally, limited courses in mathematics in early years, lack of sufficient practical exposure, poor mentoring opportunities and limited learning resources were the major limiting factors confronting the pre-service teachers in effectively adapting mathematical pedagogical content knowledge in their teaching practices.

Recommendations

Based on findings of this study, the following are the recommendations are made:

1. The low performances of PSTs in key dimensions of mathematics pedagogical content knowledge observed in this study suggest areas for improvement in teacher preparation programs. It indicates a need for enhanced focus and support in mathematics pedagogy, classroom management, communication and assessment skills.
2. Teacher preparation programs should ensure that pre-service early childhood teachers receive comprehensive training in mathematics contents and pedagogy including deep understanding of fundamental mathematical concepts, how to effectively teach mathematical concepts to young children, integrate hands on activities and manipulatives, differentiate instruction to meet diverse learning needs and developmental assessment skills required to carry out effective assessment of mathematics learning.
3. Furthermore, teacher preparation programs should provide extensive practical training and mentoring opportunities for the PSTs, this will enable them to gaining practical experience in implementing pedagogical strategies thereby empowering them to successfully navigate the complexities of teaching mathematics in the early childhood setting. Additionally, adequate learning resources for effective mathematics teaching should be provided in early childhood schools. PSTs should equally be trained on how to improvise locally available low-cost materials for improve mathematics instruction in the early years.

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