EVALUATING THE EFFECT OF SAMPLE SIZE ON BOOTSTRAP AND JACKKNIFE ESTIMATES FOR THE 2022 NABTEB MATHEMATICS MULTIPLE-CHOICE TEST ITEMS

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Abstract

The study investigated how sample size affects Bootstrap and Jackknife estimates for the 2022 NABTEB Mathematics multiple-choice test items. Using a comparative ex post facto design, data were drawn from 6,000 students who sat for the exam in South-South Nigeria, selected through multistage sampling. The validated NABTEB mathematics items served as the instrument, with a KR-20 reliability index of 0.88. R Studio software was used to compute estimates for bias, standard error, and 95% confidence intervals. Paired t-tests, ANOVA, and Cohen's d assessed differences across groups. Results showed that both methods reduced bias and standard error as sample size increased. However, the Jackknife method yielded consistently lower variability, making it more suitable for smaller samples. The Bootstrap method, with minimal resampling variability, was more effective for larger datasets. While there was no significant difference in bias estimates between the methods, the Jackknife method significantly reduced standard error. The study concluded by recommending the Jackknife method for small sample research to improve the accuracy and reliability of statistical estimates.

Keywords: Sample size, Bootstrap estimates, Jackknife estimates, Bias,Confidence interval

Introduction

Standardized testing is a cornerstone of educational assessment, serving as a basis for measuring student achievement, informing curriculum improvements and shaping educational policies. A key challenge in psychometric analysis is accurately estimating item parameters such as difficulty, discrimination and guessing indices. Traditional parametric approaches, including Classical Test Theory (CTT) and Item Response Theory (IRT), rely on large sample assumptions and normal distribution properties, which may not always hold in real-world testing scenarios. In contrast, resampling techniques such as the Bootstrap and Jackknife methods offer robust alternatives for assessing test reliability without strict distributional assumptions. These methods allow researchers to estimate bias, standard errors, and confidence intervals, making them particularly useful when dealing with small or non-normally distributed samples.

The Bootstrap method, introduced by Efron in 1979, is a Monte Carlo-based approach that generates multiple resampled datasets from the original sample with replacement, approximating the sampling distribution. The Jackknife method, initially developed by Quenouille in 1949 and later refined by Tukey in 1958, systematically omits one observation at a time to iteratively compute parameter estimates. While both techniques are widely used in psychometric evaluations, test equating, and variance estimation, their effectiveness depends significantly on sample size, raising critical questions about optimal conditions for their application in educational assessment.

Several studies have explored the implications of sample size variations on statistical estimates (Koehler et al., 2021; Van der Linden, 2020; Wang & Chen, 2023). Research suggests that while the Bootstrap method performs well with larger samples, it can introduce bias in small samples due to excessive resampling from limited data points. Conversely, the Jackknife method is often recommended for smaller datasets but may yield unstable estimates in complex test structures (Zhang & Liu, 2022). Understanding how these resampling techniques behave under different sample conditions is crucial for ensuring accurate test score interpretations and

robust psychometric properties in high-stakes assessments like National Business and Technical Examination Board (NABTEB).

Empirical studies have highlighted the trade-offs between bootstrap and jackknife resampling techniques under varying sample size conditions. For instance, recent research has demonstrated that the jackknife method tends to perform better for confidence interval estimation in small samples, whereas the bootstrap method is more advantageous for skewed distributions and larger datasets where resampling variability is less pronounced (Sroka, 2021). Similarly, DasGupta (2020) found that the jackknife method provides more stable estimates in small samples, while the bootstrap method becomes more reliable as sample size increases, particularly for skewed distributions. Furthermore, a study by Davison and Hinkley (2021) comparing these methods in estimating confidence intervals indicated that the jackknife method is more suitable for small original data samples, whereas the bootstrap method performs better for skewed distributions.

Koehler (2020) found that for sample sizes below 30, Jackknife estimates had lower bias, whereas for larger samples above 100, Bootstrap methods provided more accurate confidence intervals. Further, Kim & Wilson (2010) applied Bootstrap resampling in Item Response Theory (IRT), finding that Bootstrap-based standard errors closely matched theoretical estimates for large samples but exhibited higher variance in smaller samples. Guan et al. (2015) evaluated multiple-choice examination reliability using both methods, concluding that Jackknife estimates were more consistent for tests with fewer items, while the Bootstrap performed better for longer tests. Fan & Wang (2018) explored Bootstrap confidence intervals in test equating and found that Bootstrapmethods were more robust for non-normal score distributions. These findings reinforce the importance of sample size considerations when selecting resampling techniques for psychometric evaluations.

Despite the growing use of the Bootstrap and Jackknife in psychometric research, limited studies have examined how sample size impacts Bootstrap and Jackknife estimates in Nigerian standardized examinations. Given NABTEB's pivotal role in shaping vocational and technical education, an empirical investigation into the effect of sample size on the accuracy, stability, and reliability of these estimates is warranted. This study, therefore, seeks to analyze how

variations in sample size influence the performance of Bootstrap and Jackknife estimates for the 2022 NABTEB Mathematics multiple-choice test items. Findings from this study will inform best practices in psychometric analysis, enhancing the reliability of high-stakes testing and supporting data-driven decision-making in Nigeria's educational system.

Research Questions

The following questions were raised to guide the study.

1. How does varying the sample size influence the bias and variance of Bootstrap estimates for NABTEB's 2022 Mathematics multiple-choice items?
2. How does varying the sample size affect the bias and variance of Jackknife estimates for the same test items?
3. Which resampling method (Bootstrap or Jackknife) provides more accurate and reliable estimates for NABTEB's 2022 Mathematics multiple-choice items across different sample sizes?

Methodology

The study adopted a comparative research approach using an ex-post facto design. The population comprised all students who took the 2022 National Business and Technical Examinations Board (NABTEB) mathematics examination in South-South Nigeria. A sample of 6000 students was selected through a multistage sampling technique. The instrument used was the NABTEB 2022 mathematics multiple-choice items, which were standardized and validated by the examination body. To ensure reliability, the Kuder-Richardson Formula 20 (KR-20) was employed, yielding a reliability index of 0.88. Data collected were analyzed using R Studio software, which facilitated the computation of Bootstrap and Jackknife estimates for bias, standard error, and 95% confidence intervals. Furthermore, t-paired tests, ANOVA, and effect size calculations using Cohen's d were employed to examine differences in bias, standard error, and confidence intervals across groups.

Results

Table 1: BootstrapEstimate

Sample Size	Bias	Standard Error (SE)	95% Confidence Interval (CI)
50	-0.0030	0.0056	(0.305, 0.327)
100	-0.0010	0.0037	(0.310, 0.325)
200	-0.0013	0.0025	(0.312, 0.322)
Above 1000	-0.0007	0.0019	(0.315, 0.320)

Table 1 indicates that as the sample size increases, the bias in Bootstrap estimates decreases. At a sample size of 50, the bias is recorded as - 0.0030, but it reduces to-0.0007 when the sample size exceeds 1000. Similarly, the standard error follows a downward trend, decreasing from 0.0056 at n = 50 to 0.0019 for samples above 1000. This reduction in standard error suggests that the variability of Bootstrap estimates decreases with larger samples, leading to more stable results. Additionally, the width of the 95% confidence interval narrows with increasing sample size, reinforcing the observation that estimates become more precise with more data.

Table 2: JackknifeEstimates

Sample Size	Bias	Standard Error (SE)	95% Confidence Interval (CI)
50	-0.0029	0.0008	(0.3140, 0.3172)
100	-0.0009	0.0004	(0.3168, 0.3184)
200	-0.0014	0.0002	(0.3167, 0.3175)
Above 1000	-0.0005	0.0001	(0.3169, 0.3173)

Table 2 shows that theJackkniferesampling method shows that increasing the sample size results in reduced bias, lower variance, and improved accuracy. At a sample size of 50, the bias is -0.0029, which decreases to -0.0005 when the sample size exceeds 1000. The standard error is significantly lower in Jackknife estimates, starting at 0.0008 for n=50 and reducing to 0.0001 for larger sample sizes. Additionally, the 95% confidence intervals for Jackknife estimates are narrower, suggesting that Jackknife estimates are more precise.

Table 3: Comparison of the Bootstrap and Jackknife Methods Across Different Sample Sizes

	Bootstrap Estimates				Jackknife Estimates			
	50	100	200	>1000	50	100	200	>1000

Bias	-0.0030	-0.0010	-0.0013	-0.0007	-0.0029	-0.0009	-0.0014	-0.0005
Standard Error (SE)	0.0056	0.0037	0.0025	0.0019	0.0008	0.0004	0.0002	0.0001
95% Confidence Interval (CI)	(0.305, 0.327)	(0.310, 0.325)	(0.312, 0.322)	(0.315, 0.320)	(0.305, 0.327)	(0.314, 0.317)	(0.317, 0.318)	(0.317, 0.318)

Table 3 reveals that the Jackknife method consistently provides more accurate and reliable estimates than the Bootstrap. The bias in Jackknife estimates is lower across all sample sizes, and the standard error is considerably smaller, indicating that Jackknifeestimates are more stable. Moreover, the confidence intervals for Jackknife estimates are narrower across all sample sizes, suggesting that Jackknife resampling provides greater precision in estimating test parameters. This makes the Jackknife method a preferable choice when high accuracy is required. However, Bootstrap resampling remains useful, especially in cases where computational efficiency is a concern or when working with smaller datasets.

Table 4: Statistical Evaluation of Bias, Standard Error (SE), and Effect Sizes

Test	Bias (p-value)	Standard Error (SE) (p-value)
Paired t-test	0.3189 (not significant)	0.0192 (significant)
ANOVA	0.9221 (not significant)	0.0104 (significant)
Levene's Test	0.8939 (not significant)	0.0716 (not significant)
Cohen's d	-0.0721 (small effect)	2.5971 (large effect)

Table 4indicates that the bias difference between the two methods is not statistically significant ($p = 0.3189$), suggesting that both resampling techniques produce comparable bias levels. However, the difference in standard error is significant ($p = 0.0192$), implying that one method exhibits significantly lower variability than the other. Table 4 furthershows that bias differences are not statistically significant ($p = 0.9221$), reinforcing the finding that both resampling methods produce similar bias values. However, the standard error difference is significant ($p = 0.0104$), indicating that variability in estimates differs depending on the resampling method or sample size. Levene's test was conducted to assess whether the variance in bias, and standard error, is equal across different sample sizes. The test results indicate that none of the three measures show significant differences in variance ($p = 0.8939$ for bias, and $p = 0.0716$ for

standard error). This suggests that the variability in resampling estimates remains relatively consistent across different sample sizes.

Furthermore, Cohen's d was used to measure the effect size of differences in bias and standard error between the resampling methods. The bias difference yielded a small effect size (d = -0.0721), indicating a negligible practical difference between methods. However, the effect size for standard error was large (d = 2.5971), signifying a substantial difference in variability between Bootstrap and Jackknife estimates. The statistical analyses indicate that bias differences between Bootstrap and Jackknife estimates are not significant, implying that both methods provide similar levels of bias. However, the Jackknife method demonstrates significantly lower standard error, as reflected in both the paired t-test and ANOVA results, along with a large effect size for standard error. This suggests that Jackknife estimates are more stable and less variable than Bootstrap estimates. Additionally, Levene's test results indicate that the variance in estimates does not significantly differ across sample sizes. Overall, while both resampling methods offer comparable bias levels, the Jackknife method is preferable due to its lower standard error and potential accuracy advantage.

Discussion of Findings

The findings of this study indicate that the Bootstrap method demonstrates a decreasing bias trend as sample size increases. Similarly, the standard error (SE) declines with larger sample sizes, suggesting improved estimate stability. The progressive narrowing of the 95% confidence interval further supports the notion that Bootstrap resampling yields more precise estimates as dataset size increases. These findings align with recent research (Koehler, 2020; Zhang & Li, 2021), which suggests that while Bootstrap resampling is effective with large samples, it may introduce bias when applied to small datasets due to excessive reliance on limited data points.

Similarly, the Jackknife method exhibits a comparable pattern of decreasing bias and standard error with increasing sample size. Notably, the standard error in Jackknife estimates remains significantly lower than that of Bootstrap estimates, suggesting that the Jackknife method provides more stable and reliable parameter estimates. This supports previous findings (Wang

et al., 2019; Liu & Zhao, 2021), which indicate that the Jackknife method is particularly advantageous for small-sample applications due to its reduced variability.

Furthermore, this study highlights that the Jackknife method consistently produces more accurate and precise estimates compared to the Bootstrap method. Across all sample sizes, bias in Jackknifeestimates remains lower, and standard error is significantly smaller, reflecting greater stability. The narrower confidence intervals further support the assertion that Jackknife resampling improves precision in parameter estimation. While Bootstrap resampling remains valuable, particularly for computational efficiency and large datasets, the Jackknife method appears superior in minimizing estimation variability, especially when dealing with small samples. These findings align with prior research (Chen et al., 2016; Park & Kim, 2022), which suggests that Jackknife resampling is more effective in small samples, whereas Bootstrap methods perform better with larger datasets.

Additionally, the findings reveal that the difference in bias between the two resampling methods is not statistically significant, indicating that both techniques produce comparable levels of bias. However, the difference in standard error is statistically significant, demonstrating that one method exhibits substantially lower variability than the other. The Cohen's d effect size for standard error is large, reinforcing the substantial difference in variability between Bootstrap and Jackknife estimates. These results are consistent with recent studies (Fan & Wang, 2018; Zhou et al., 2023), which found that Jackknife estimates tend to exhibit lower variability than Bootstrap estimates.

Overall, this study suggests that while both Bootstrap and Jackknife resampling methods provide valuable insights, the Jackknife method is preferable for smaller sample sizes due to its lower standard error and greater stability. Conversely, the Bootstrap method is more effective for larger datasets, where the impact of resampling variability is reduced. These findings align with empirical evidence from recent studies (Zhang & Liu, 2022; Wang & Chen, 2023), emphasizing the importance of considering sample size when selecting resampling techniques in psychometric evaluations.

Conclusion

Based on the findings of this study, it was concluded that while both the Bootstrap and Jackknife methods exhibit decreasing bias and standard error with increasing sample size, the Jackknife method consistently demonstrates lower variability, making it a more stable option, particularly for small samples. In contrast, the Bootstrap method remains advantageous in larger datasets, where resampling variability has a minimal impact. The study also establishes that, although the bias difference between the two methods is not statistically significant, the difference in standard error is substantial, favouring the Jackknife method for more precise estimations.

Recommendations

Based on the study findings, it was recommended that:

1. Researchersconducting studies with small sample sizes should prioritize the Jackknife resampling method to minimize estimation variability and improve the reliability of parameter estimates.
2. For studies involving larger datasets, the Bootstrap method remains a viable option due to its computational efficiency and ability to generate precise estimates with a sufficiently large sample.

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